ABC Hoops Metrics: An Evaluation of a Player's Skill Using Holistic Measures

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Abstract

The purpose of this paper is to develop an evaluation tool for players in ABC Hoops. The conventional basketball statistics that are recorded each game (points, rebounds, assists, rebounds, fouls) is a relatively new phenomenon for recreational sports leagues. Having been one of the first leagues to begin tracking statistics, ABC Hoops has always prided itself with providing our players with a unique and highlighted aspect of the game. While the amount of points a player scored per game is a valuable indicator of his or her talent, we conceded that many external factors can be involved. Perhaps player evaluation should be better judged using assists or rebounds. We decided to take these and many other factors into account and apply various statistical techniques with our current numbers to judge a basketball player holistically. We feel the equation derived is not a perfect solution but certainly a valid indicator of a player's worth. Many of these measures have been derived from Dean Oliver, a prominent statistician in the movement of basketball analytics.

1 Pythagorean Expectations

1.1 Overview

Bill James, the pioneer behind Sabermetrics, developed the Pythagorean Expectation formula to estimate the probability a baseball team wins given the average number of runs a team scores and allows. The original equation is given by:

$$WinPercentage = \frac{RS^{\lambda}}{RS^{\lambda} + RA^{\lambda}} \tag{1}$$

where RS denotes runs scored, RA denotes runs allowed, and λ is any exponent. The equation was coined as 'The Pythagorean Equation' because early estimates of λ showed to be 2. In the sport of basketball, runs scored (RS) is replaced by points forward (PF), which is the total amount of points a team scores, and runs allowed (RA) is replaced by points allowed (PA), or the total points the opponents have scored. In order to approximate λ for ABCHoopsNYC, we incorporate statistics over nine seasons to fit our data to James equation. We assigned the Pythagorean measure, or the expectations of a team winning a weight of 3%, given that we are concerned with individual worth rather than team.

1.2 Theory

We begin our model with a Weibull distribution function given by

$$f(x, \lambda, k) = \begin{cases} \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}e^{-(\frac{x}{\lambda})^{k}} & : x \geq 0 \\ 0 & : x < 0 \end{cases}$$

where k denotes the shape parameter and λ denotes the scale parameter of the distribution. Following Steven Miller's derivation, we include a translation parameter β to account for the discrete nature of our data in PF and PA. In a typical ABCHoopsNYC game, total scores range from about 30-80. As a result, the pdf can be rewritten as

$$f(x, \lambda, k) = \begin{cases} \frac{k}{\lambda}(\frac{x-\beta}{\lambda})^{k-1} e^{-(\frac{x-\beta}{\lambda})^k} & : x \geq 0 \\ 0 & : x < 0 \end{cases}$$

We define a function

$$\Gamma(s) = \int_0^\infty e^{-u} u^{s-1} du \tag{2}$$

Or equivalently,

$$\Gamma(s) = \int_0^\infty e^{-u} u^s \frac{du}{u} \tag{3}$$

The mean of the Weibull Distribution is given by

$$\mu = x\Gamma(s) \tag{4}$$

$$\mu = \int_\beta^\infty x \frac{k}{\lambda}(\frac{x-\beta}{\lambda})^{k-1} e^{-(\frac{x-\beta}{\lambda})^k} dx \tag{5}$$

Subtracting β from inside the integral and adding it outside of the expression gives us:

$$\mu = \int_\beta^\infty \lambda(\frac{x-\beta}{\lambda}) \frac{k}{\lambda}(\frac{x-\beta}{\lambda})^{k-1} e^{-(\frac{x-\beta}{\lambda})^k} dx \tag{6}$$

By methods of u-substitution, we set $u = (\frac{x-\beta}{\lambda})^k$; thus du = $\frac{k}{\lambda}$ $(x\text{-}\beta)^{k-1}$

$$\mu = \int_0^\infty \lambda^k \sqrt{u} e^{-u} du + \beta \tag{7}$$

$$\mu = \lambda \int_0^\infty e^{-u} u^{1+k^{-1}} \frac{du}{u} + \beta \tag{8}$$

$$\mu = \lambda\Gamma(1 + k^{-1}) + \beta \tag{9}$$

Let X and Y be independent random variables with Weibull Distributions, where X is points forward, and Y is points allowed. From Equation (9), the mean for PF and PA can be expressed as

$$PF = \lambda_{PF}\Gamma(1 + k^{-1}) + \beta \tag{10}$$

$$PA = \lambda_{PA}\Gamma(1 + k^{-1}) + \beta \tag{11}$$

$$\lambda_{PF} = \frac{PF - \beta}{\Gamma(1 + k^{-1})} \tag{12}$$

$$\lambda_{PA} = \frac{PA - \beta}{\Gamma(1 + k^{-1})} \tag{13}$$

2 Rebound Rate

The statistics on rebounds found on the ABCHoopsNYC website is separated between offensive and defensive rebounds. The act of rebounding, especially in a matchup with bigger players requires aggressiveness and ball awareness. We decided to take into account a player's rebounds as a percentage of his or her team. This not only measures a player's effective rebound, but also how much of a team's total rebound the player is responsible for. The rebound rate for player i is given by RR_i.

$$RR_i = \frac{Rebounds_i}{TeamRebounds} \tag{14}$$

3 Effective Assists

We define as assist to be made when a player passes a basketball to another player who scores a shot immediately or three steps after. Here at ABCHoopsNYC, we are more than generous when it comes to assist. By definition, one unit of assist is equivalent to one field goal. While an assist can be a valid measure of a player's ability to share the ball and help the team, we can assist to be a percentage of the total field goal a team makes. We define this percentage as an effective assists (EA).

$$EA_i = \frac{TotalAssists_i}{TeamFieldGoal} \tag{15}$$

4 Free Throw Percentage

A free throw opportunity is awarded to a player who is fouled in the act of shooting or when the opponent's total team fouls exceeds a threshold. In ABCHoopsNYC, this threshold, or bonus, occurs when a team's total fouls adds to 7 or 10. A free throw percentage can be easily computed by dividend the total free throws made by the total free throws attempted.

$$FT_i = \frac{TotalFTi}{TotalFT} \tag{16}$$

5 Defensive Measure

A defensive measure is defined as the sum of steals, blocks, and fouls. These categories typically determine a player's defensive ability while isolating one's offensive scores.

$$DM_i = Steals_i + Blocks_i + Fouls_i \tag{17}$$

6 Approximate Value

The notion of approximate value takes into account most basketball statistics in a game. We begin by defining Credits as the following:

$$Credit_i = Points_i + Rebounds_i + Assists_i + Steals_i + Blocks_i - FG - Missed_i - FT - Missed_i \tag{18}$$

The number of credits provides a crude approximation to how a player is valued. In order to account for outliers in statistics i.e. a player having an extraordinary number of rebounds, FG missed, etc., we scale the value of credits in the following manner:

$$AV_i = \frac{Credits^{3/4}}{21} \tag{19}$$

The scaling factor is derived from Martin Manley, in which he uses the technique of linear methods to explain this factor. In ABCHoopsNYC, we define this number as approximate value.

7 Points Per Game

The last independent variable of ABC+ is points per game, defined as the total amount of points player i scores in a season divided by the total amount of games. This variable captures how consistent and reliable a player each game.

$$PPG_i = \frac{TotalPoints_i}{NumberofGames} \tag{20}$$

8 Holistic Model

Combining the previous sections, we have the following multiple regression model.

$$ABC+_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 + \epsilon \quad (21)$$

x_1	Pythagorean Expectation	$\beta_1 = .03$
x_2	Rebound Rate	$\beta_2 = .05$
x_3	Effective Assist	$\beta_3 = .05$
x_4	Free Throw Percentage	$\beta_4 = .07$
x_5	Defensive Measure	$\beta_5 = .12$
x_6	Approximate Value	$\beta_6 = .3$
x_7	Points Per Game	$\beta_7 = .03$

References

[1] Miller, Steven J., *A Derivation of the Pythagorean Win-Loss Formula in Baseball*. ArXiv Mathematics e-print, 2005.

[2] Oliver, Dean., *JoBS Methods Index*. Journal of Basketball Statistics, http://www.rawbw.com/ deano/